

We denote by x_i the measured random observable x in the i -th event and by $(w_x)_i$ the observable's weight in that event. Even if the p.d.f. $f(x)$ is completely unknown we can still use measured values x_i to estimate mean and variance of a random variable x . In particular, the unbiased estimator for mean is given by

$$\langle x \rangle \equiv \frac{\sum_{i=1}^N (w_x)_i x_i}{\sum_{i=1}^N (w_x)_i}, \quad (1)$$

and the unbiased estimator for variance is given by

$$s_x^2 \equiv \left[\frac{\sum_{i=1}^N (w_x)_i (x_i - \langle x \rangle)^2}{\sum_{i=1}^N (w_x)_i} \right] \times \left[\frac{1}{1 - \frac{\sum_{i=1}^N (w_x)_i^2}{[\sum_{i=1}^N (w_x)_i]^2}} \right]. \quad (2)$$

The final results and statistical errors of a random variable x are then reported as:

$$\langle x \rangle \pm \frac{\sqrt{\sum_{i=1}^N (w_x)_i^2}}{\sum_{i=1}^N (w_x)_i} s_x. \quad (3)$$

Unit weights. For the case of unit weights, $(w_x)_i = 1$, the above equations simplifies into:

$$\langle x \rangle \equiv \frac{\sum_{i=1}^N x_i}{N}, \quad (4)$$

$$s_x^2 \equiv \frac{\sum_{i=1}^N (x_i - \langle x \rangle)^2}{N - 1}, \quad (5)$$

$$\langle x \rangle \pm \frac{s_x}{\sqrt{N}}. \quad (6)$$